

A REVIEW OF TETHER INDUCED DYNAMICAL FEATURES

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INTRODUCTION

A talk in the general field of tether fundamentals cannot be started without a mention to the S.A.O. report of september 74 {1} where the "Skyhook" (this was the first name given to the TSS) was presented. In fact, as pointed out in {2} and {3}, it is true that tethers in space have been conceived since the last years of the 19-th century, but it is also true that until the 70-s they had been part of more or less visionary concepts. On the contrary, in {1} the idea was put on sound engineering grounds and the compatibility with the Space Shuttle was clearly shown. Consequently, investigations of possible uses of tethers in space were undertaken and in the succeeding years the peculiarities of TSS motion were investigated extensively, so that at present it can be said that, if elastic effects are ignored, TSS dynamics is sufficiently well known. Further, it can also be said that the experience acquired in past and present investigations is sufficient to allow the simulation of the motion of more complex tethered systems with a reasonable degree of accuracy.

SOME FEATURES OF TETHER DYNAMICS

In order to make a review of the most peculiar features of tether dynamics let us consider the simplest mathematical model having been used for the simulation of TSS motion. However, as it will appear from the assumptions below, the same model is also useful to investigate a larger class of tethered platforms.

In fact, most of the systems proposed so far for future applications have mechanical features (mass, inertia moments, orbit) and operational requirements which are largely different from those of the first satellite, but the relevant environmental forces (gravity gradient, Coriolis during manoeuvres, eccentricity excitation, etc.) will be the same.

Let us assume that:

- the system is composed by a massive main body (space station or other) in circular orbit and by a smaller platform connected to it by means of a variable length tether; moreover, the mass of the tether is negligible;
- the Earth is spherical and homogenous, so that oblateness and higher order gravitational perturbations are ignored;
- non gravitational forces, as aerodynamic drag or other elasticity effects are very small so that they can be neglected;
- the platform is a point mass.

In this case, it is well known that the system has two stable equilibrium configurations, aligned with the local vertical, so that the classical methods of mechanical vibrations can be used to investigate the motion of the platform in the neighbourhood of them. Therefore, if the gravitational energy is reduced to a quadratic form with respect to the ratio l/a of the tether length to the semimajor axis of the station orbit and if the small amplitude approximation is also made, the dynamical equations can be written as:

$$\ddot{\theta} + 2 \frac{\dot{\ell}}{\ell} \dot{\theta} + 3n^2 \theta = -2n \frac{\dot{\ell}}{\ell} \quad (1)$$

$$\ddot{\phi} + 2 \frac{\dot{\ell}}{\ell} \dot{\phi} + 4n^2 \phi = 0 \quad (2)$$

where:

- θ is the offset angle from local vertical in the orbit plane
- ϕ is the out of plane offset angle
- n is the orbit mean motion
- the dots mean differentiation with respect to time.

First, let us consider station keeping conditions, where tether length is constant. From (1) and (2) it is immediate to verify that motion is stable, consisting of two uncoupled librations with constant amplitudes. It can be seen from fig.1 that tether periods are slowly increasing functions of the altitude h .

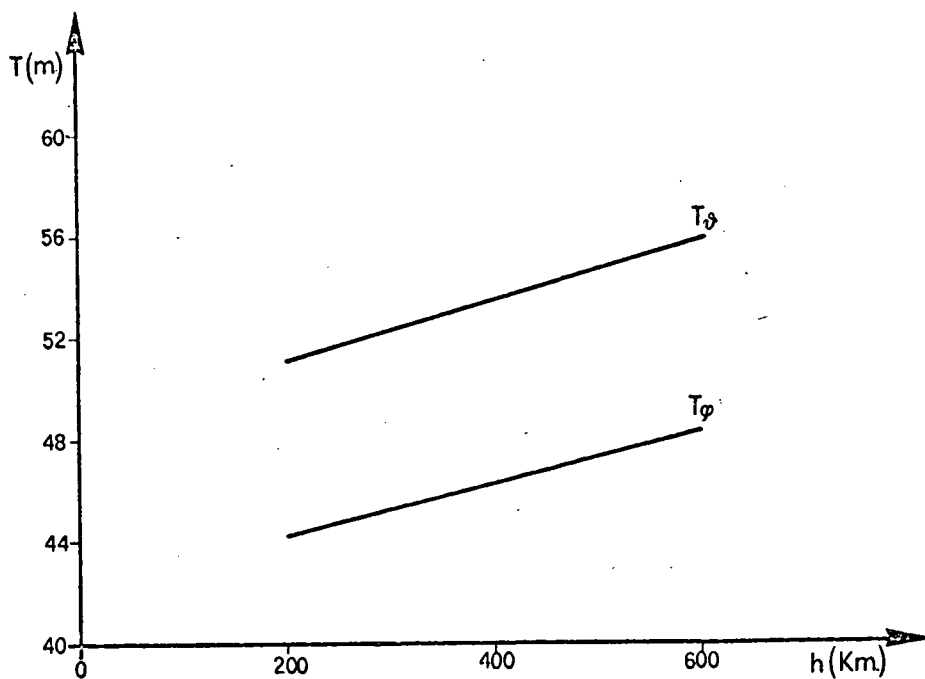


FIG. 1. -

If tether length is not constant in time, as during deployment and retrieval, the terms proportional to \dot{l} in eqs.(1) and (2) are different from zero. First, let us consider the last (forcing) term in the θ equation; its structure suggests that it is originated by the Coriolis force which pushes the platform away from the local vertical. In plane instability can occur during the first phase of deployment and the last of retrieval; if the same control law for l is used in the two phases, they are equally critical. The asymmetry between deployment and retrieval is apparent from the velocity dependent terms in both the equations: it is seen that when the coefficients are positive (i.e.during deployment) librations are damped by length increase, on the contrary, during retrieval self excited librations can occur, so that the most critical situation is encountered in the last phase of retrieval.

ELASTICITY EFFECTS

The considerations made so far have ignored elasticity effects. Unfortunately, (from the point of view of simulation problems), the tether is an elastic continuum, so that it can undergo a variety of vibrations: longitudinal, torsional, lateral (both in plane and out of plane). A preliminary evaluation of the frequencies involved in the TSS case has been made in {4} and {5} and further investigation is currently in progress. The same analysis can be extended to different tethered systems, but since now it can be expected that the

frequencies of elastic modes will be much higher than those of pendulum-like librations. This is a well known feature of TSS dynamics; its occurrence is the cause of serious difficulties in simulating the motion with purely numerical methods. In fact the integration step must be small enough to allow correct simulation of the short period component of the motion, so that the time needed for a physically meaningful numerical simulation can easily be excessive.

A major problem is the evaluation of the dynamical noise acting on the platform. In fact, one appealing feature of tethered platforms is the possibility of attaining high pointing accuracies by isolating them from the noise originated in the primary. To make an example, Aeritalia has investigated the possibility of actively controlling the motion of the point of attachment of the tether to the SATP (Science and Application Tethered Platform) in order to achieve a pointing accuracy of the order of 1 arcsec in attitude control. In this frame the tether itself can be viewed as a passive damper the efficiency of which must be tested carefully.

This is because at present very scarce information is available even about the properties (in particular about structural damping) of the materials to be possibly used in the first TSS flights. It is expected that post flight analysis of the accelerations at the satellite will provide some informations on tether damping, but the tuning (if possible) of a tether to a given system, in order to maximize energy dissipation, will certainly require further experiments.

One additional problem is the possibility of coupling between the attitude motion of the platform and some of the higher modes of tether lateral vibrations. While coupling can be expected on the basis of approximate modal analysis, a reasonable estimate of vibration amplitudes requires the knowledge of excitation sources and system damping. Again, at present experience is lacking, so that the analysis on TSS will be useful as a first step to understand more complex systems and, in particular, to discriminate between what is really important from what is negligible (at different levels of accuracy).

Perturbation sources

In spite of the problems mentioned above, a preliminary knowledge of the response of a tethered system to most likely perturbations is fundamental, in order to make an evaluation of the order of magnitude of the dynamical noise to which it is expected to be subject.

What follows is a tentative list of the best known mechanical perturbing actions; from the comparison of their dynamical features with system natural frequencies it is possible to have a feeling of their impact on the motion and, consequently, on experiments requirements.

Orbit eccentricity

Nominally, the orbit of the Space Station (S.S.) will be circular, but the actual orbit will be allowed a resi-

dual eccentricity e . In a pendulum-like system e affects the in plane motion originating a forced libration with amplitude equal to e and period equal to the orbital period. With $e = 10^{-4}$ and a tether length of 10 Km, the acceleration amplitude would amount to $\approx 1.2 \cdot 10^{-7}$ g.

Earth oblateness

The well known secular regression of the line of nodes and advance of the apsidal line should not cause major dynamical problems to a tethered system. However, perhaps it is less widely known {6} that Earth oblateness causes both semimajor axis a and inclination i of a circular orbit to undergo variations with periods equal to half the orbit period, i.e. with frequency equal to the out of plane libration frequency. Therefore resonance can occur and vibration amplitudes can increase in time.

The mathematical modelization of this dynamical feature is not simple. At present, it is believed that, due to the relatively short time span of the mission, TSS libration amplitudes cannot grow to undesired levels; however further investigation is needed if longer missions, in connection to the S.S., are envisaged.

Temperature changes

Tether length in the stressed equilibrium configuration parallel to the local vertical depends on temperature. In low altitude, low inclination orbits, space tempera-

ture changes by some 10 deg. twice per orbit, so that the possibility exists that longitudinal vibrations be excited during the transition from sunlight to Earth shadow and vice-versa.

The maximum acceleration at the platform depends on the coefficient of thermal expansion α of the tether. Testing of candidate materials should take into account also this aspect of the problem; in fact it does not seem impossible, in line of principle, to use a tether with a very small α value.

Internal sources of perturbation

One of the features of platforms tethered to the S.S. is the possibility to act as almost independent subsystems with minimum interference with other S.S. activities. The tether, however can transmit disturbances to the platform; in this way noise can be originated by Shuttle docking, station keeping manoeuvres, crew motion, etc.

At present, no reliable estimate is possible of the dynamical noise at the platform, because of lack of information on tether damping properties. In this concern, as mentioned before, the results of a study about the possibility of using the tether as a structural damper could pay for the effort.

Natural frequencies

From the review above it can be seen that perturbing forces can be categorized into two groups with respect to the frequency of the excitation. Long period forces are those with frequencies comparable to the mean motion (eccentricity and Earth oblateness effects) while short period forces are those with much higher frequencies (at least one order of magnitude). The former ones are not likely to excite tether elastic modes, while the latter, on the contrary, can do that. This is the reason why a numerical example of the frequencies possibly involved is presented below in a study case.

Let us assume that the orbit is circular at 500 Km height and that the platform mass is $m: 5 \cdot 10^4$ Kg. Also, the tether parameters are:

$$\rho = 1.5 \text{ Kg/m}^2 \quad \text{mass density}$$

$$\mu = 0.5 \text{ Kg/m} \quad \text{mass per unit length}$$

$$E = 7 \cdot 10^{10} \text{ N/m}^2 \text{ Young modulus}$$

Longitudinal vibrations

If it is assumed that the tether end at the S.S. is fixed and that, at the platform, the inertia force must be balanced by the elastic stress, approximate values of the periods of the first longitudinal vibration modes can easily be found {7}.

The first five periods are reported in Tab.1 for two different tether lengths.

T(sec) \ λ (Km)	1	10
T 1	9.2	29.5
T 2	0.29	2.9
T 3	0.15	1.5
T 4	0.10	0.98
T 5	0.07	0.7

TAB. 1. -

Lateral vibrations

The same approach can be adopted for lateral vibrations. The main difference is that in the fundamental modes elasticity is not relevant and that in plane and out of plane librations have different periods. On the contrary, elastic effects are dominant in upper modes, so that higher frequencies are almost coincident. For this reason, only the periods from T 2 to T 5 are reported in Tab.2.

T(sec) \ λ (Km)	1	10
T 2	141	306
T 3	70	156
T 4	47	105
T 5	35	79

TAB. 2. -

Torsional vibrations

Torsional vibrations are believed to have a minor impact on the overall noise, because of the very large ratio between platform yaw moment of inertia and tether inertia. Only the torsional spring mass mode could be excited by disturbances originated in the platform itself. No evaluation is made of the corresponding period because of the uncertainty of the parameters involved.

The period of the upper modes is, however, in the range of 1 sec or less for both the lengths.

Platform attitude motion

The evaluations of the periods is quite uncertain, because of the large variability range of the parameters involved. However, let us assume that the distance of the platform c.o.g. from the tether attachment point be equal to 5 m and that the radius of gyration both in pitch and in roll be 3 m. The periods are shown in Fig.2 vs.tether length.

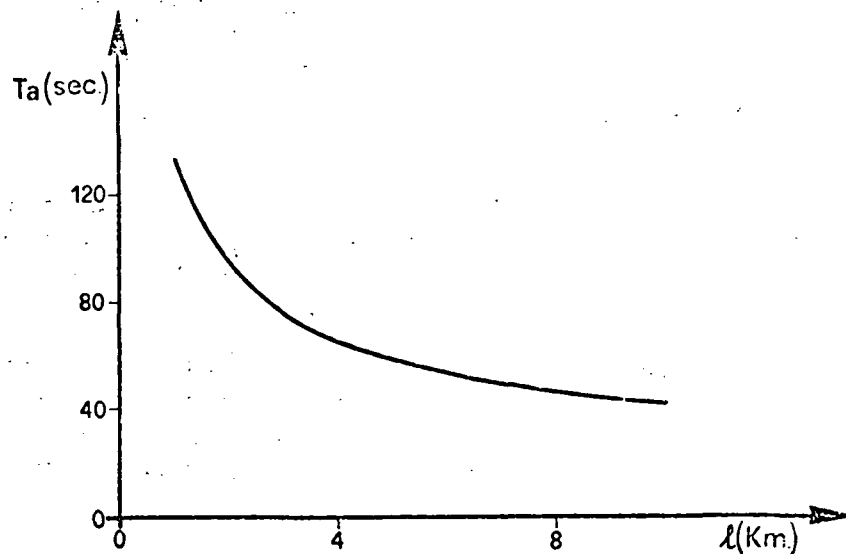


FIG. 2. -

CONCLUSIONS

Tethered systems provide a very interesting and, in some sense, unique opportunity for scientific activity in space. Some of the experiments envisaged so far, however, require the measurement of very small mechanical quantities (accelerations etc.). This implies that the level of dynamical noise on instruments output be low or that system response to excitations, either external or internal, be sufficiently known.

In this respect, the first TSS flights will be very useful, but much work will be needed in order to have reliable estimates of structural damping in different future systems.

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